Gernot Hoffmann

Circle a Square

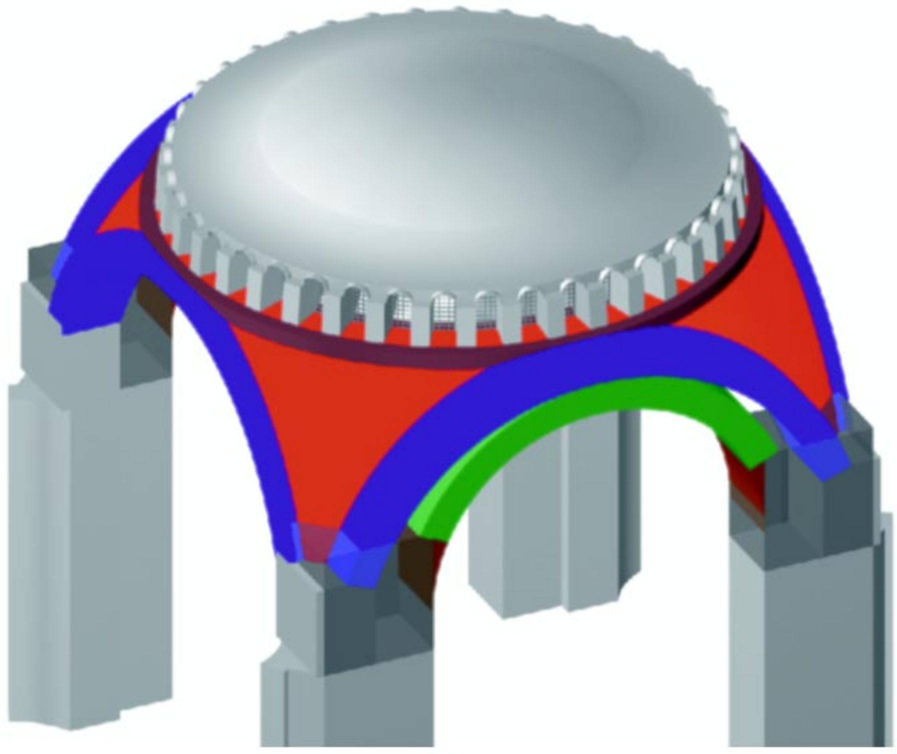


Table of Contents

1. Introduction

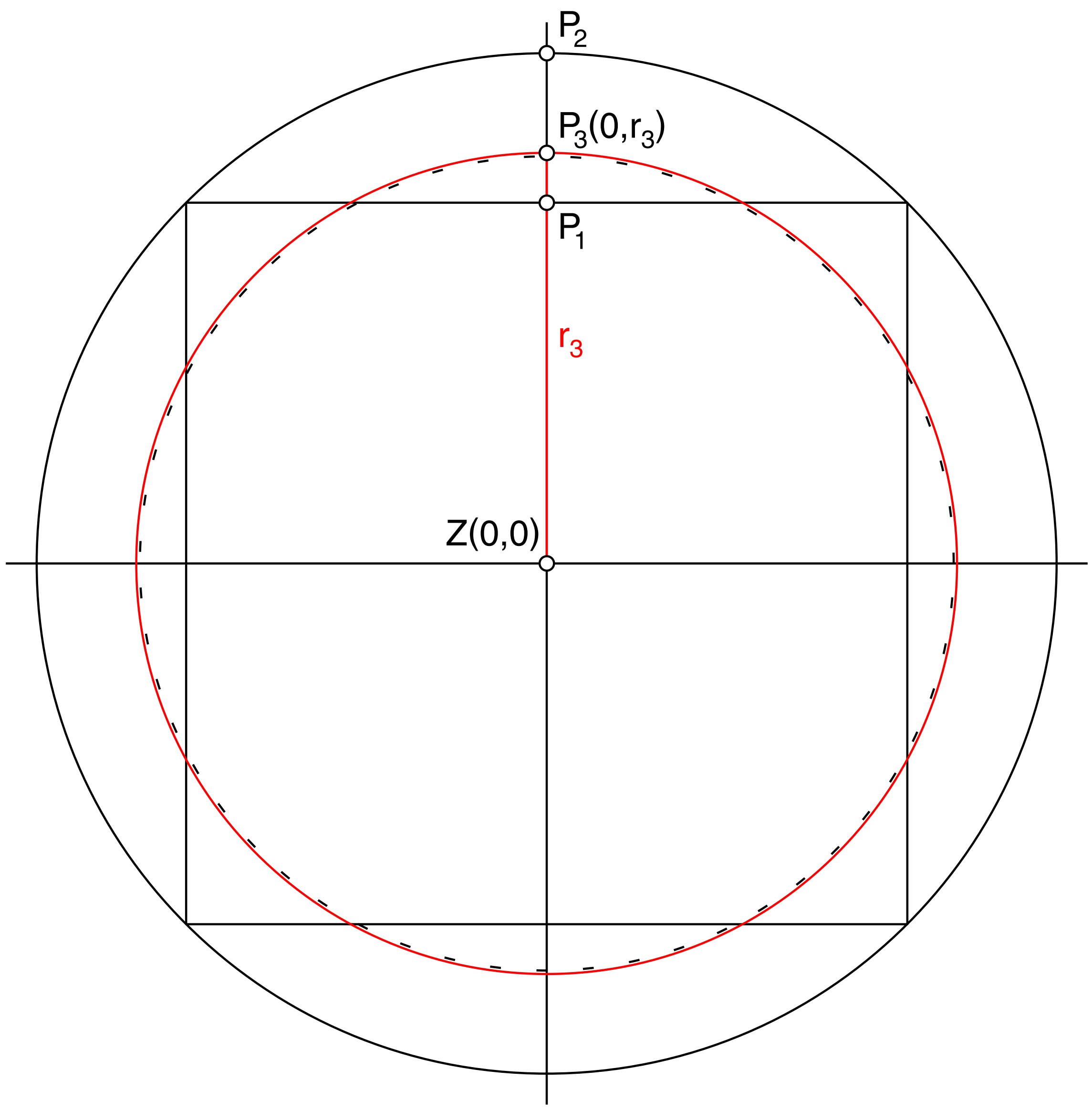
‘Circle a square’ is the inverse problem to the classic task ’square a circle’: find the radius of a circle which has the same area as a square.

This doc shows four methods. Method A and method B as published by Volker Hoffmann on p.39 of this exhibition catalogue [1]. Method A is very old, it is hardly possible to date the first publication.
Unfortunately, the accuracies of the approximations were indicated wrong by a factor of hundred (percentage). The new method B itself is rather accurate though, as proved by the mathematical derivation.
Method C is not as simple as method B and less accurate.
Method D is the most accurate solution, by courtesy of Mr.R.S.J.Reddy [4].

2. Method A

The square has the edge length two units.

1. Draw the black circle about the center Z
2. Divide P_1,P_2 by factor 3, using compass and ruler
3. Find P_3
4. The red circle about Z through P_3 with radius r_3 is the circled square
   The black dashed circle shows the accurate solution



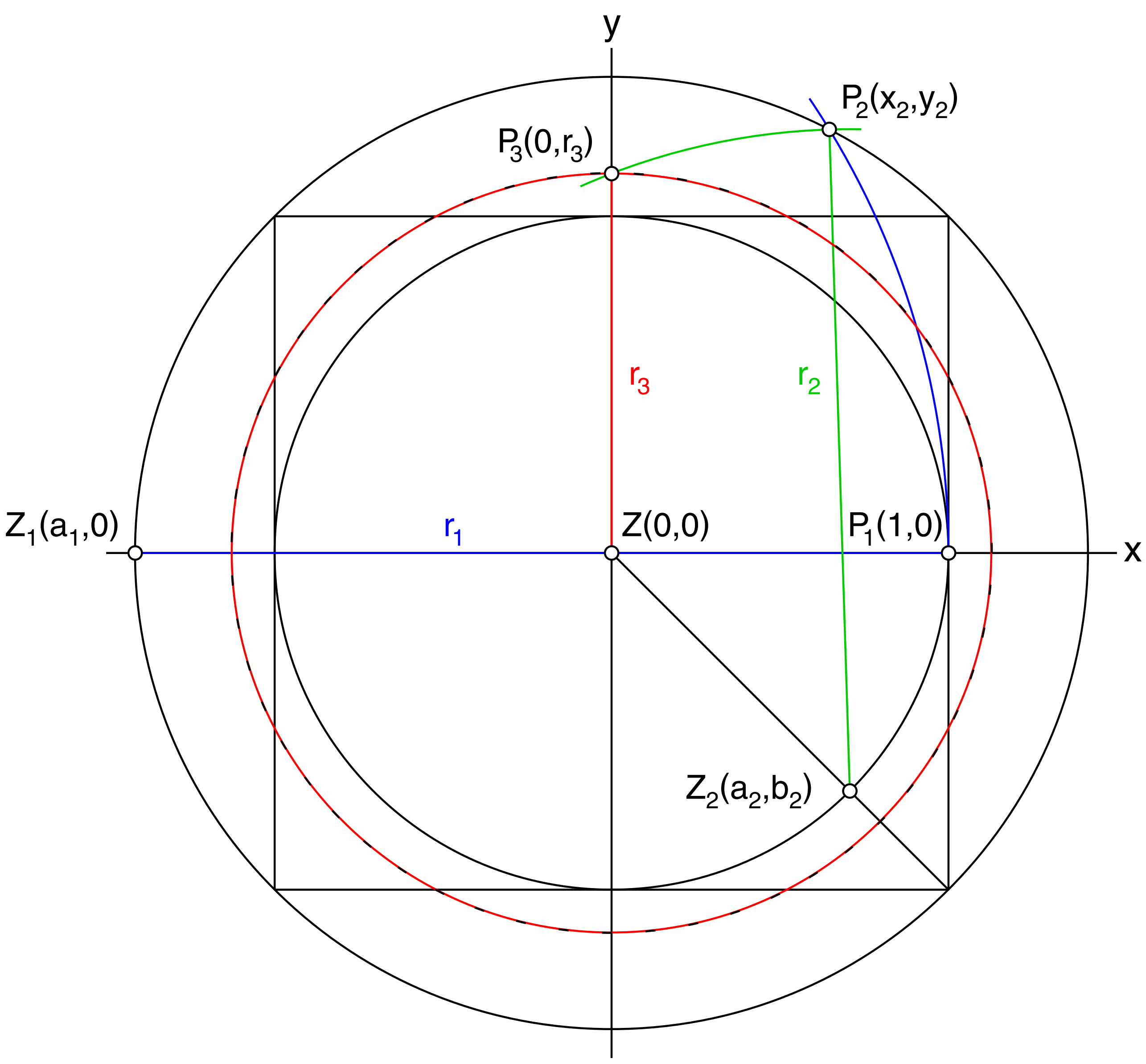
Square-Area / Circle-Area = 0.98304016, equivalent to an accuracy of 1.7 %

3. Method B

This is the method by Volker Hoffmann and Nikolaos Theocharis.

The square has the edge length two units.

1. Draw the two black circles about the center Z
2. Draw the blue circle about Z_1 through P_1
3. Find the intersection P_2
4. Draw the green circle about Z_2 through P_2
5. Find the intersection P_3 at $x=0$
6. The red circle about Z through P_3 with radius r_3 is the circled square
   The black dashed circle shows the accurate solution



Square-Area / Circle-Area = 1.00212785, equivalent to an accuracy of 0.21%

4. Method B / Mathematics

The calculation is straightforward. The intersection of two overlapping circles delivers two solutions. The proper one was chosen.

Edge length of the square: 2 units

$$a_1 = -\sqrt{2}$$
$$r_1 = +\sqrt{2} + 1$$

Black and blue circle:

$$x^2 + y^2 = a_1^2$$
$$(x - a_1)^2 + y^2 = r_1^2$$

Subtract first equation from second and solve for x_2,y_2:

$$x_2 = \frac{2a_1^2 - r_1^2}{2a_1}$$
$$y_2 = \sqrt{a_1^2 - x_2^2}$$

Green circle:

$$a_2 = +\sqrt{2}/2$$
$$b_2 = -\sqrt{2}/2$$

$$(x - a_2)^2 + (y - b_2)^2 = r_2^2$$
$$r_2^2 = (x_2 - a_2)^2 + (y_2 - b_2)^2$$

$$x_3 = 0$$
$$y_3 = r_3$$

$$(-a_2)^2 + (r_3 - b_2)^2 = r_2^2$$

Red circle:

$$r_3 = b_2 + \sqrt{r_2^2 - a_2^2}$$

Ratio of areas:

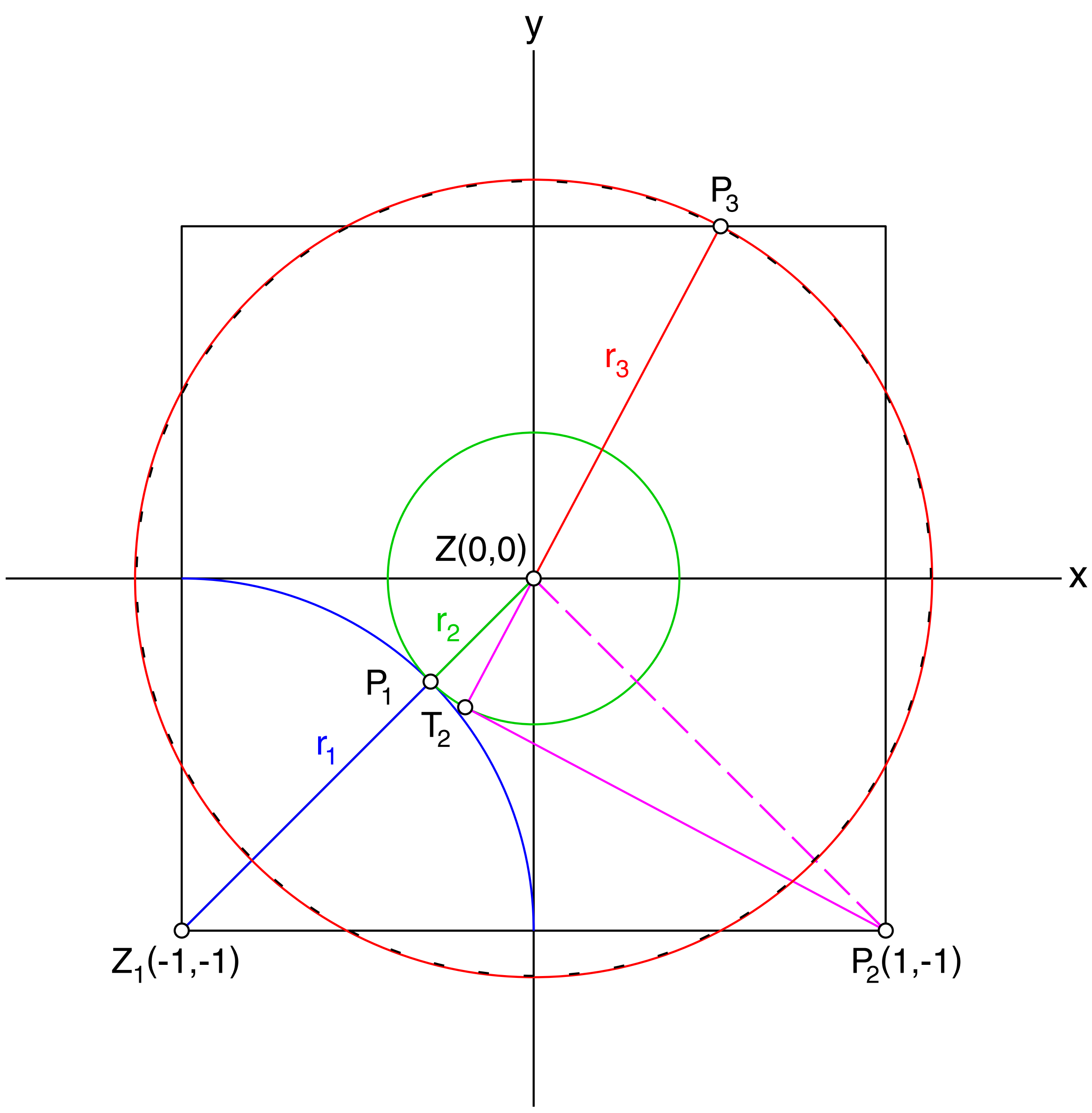
$$\text{Square}/\text{Circle} = 4/(\pi r_3^2)$$

5. Method C

This method by Tan Tai Nguyen [3] is an approximation as well. The author seems to believe that his solution is accurate.
Neither for ‘circling a square’ nor for ‘squaring a circle’ an accurate solution by ruler and compass exists.

The square has the edge length two units.

1. Draw the blue circle about Z_1
2. Draw the diagonal and find P_1
3. Draw the green circle about Z through P_1
4. Find the tangent point T_2 from P_2 for the green circle (*Thales* circle over P_2,Z)
5. Draw the line through T_2 and Z and find the intersection P_3
6. The red circle about Z through P_3 with radius r_3 is the circled square
   The black dashed circle shows the accurate solution
   Note: the radius r_3 is the longer part of T_2,P_2, intersected at x=0, as well.



Square-Area / Circle-Area = 0.99318851, equivalent to an accuracy of 0.68%

6. Method D

This method by R.S.J.Reddy [4] is an approximation as well. The author wants to replace the common value for π by an algebraic irrational expression π'.

$$\pi = 3.14159265$$

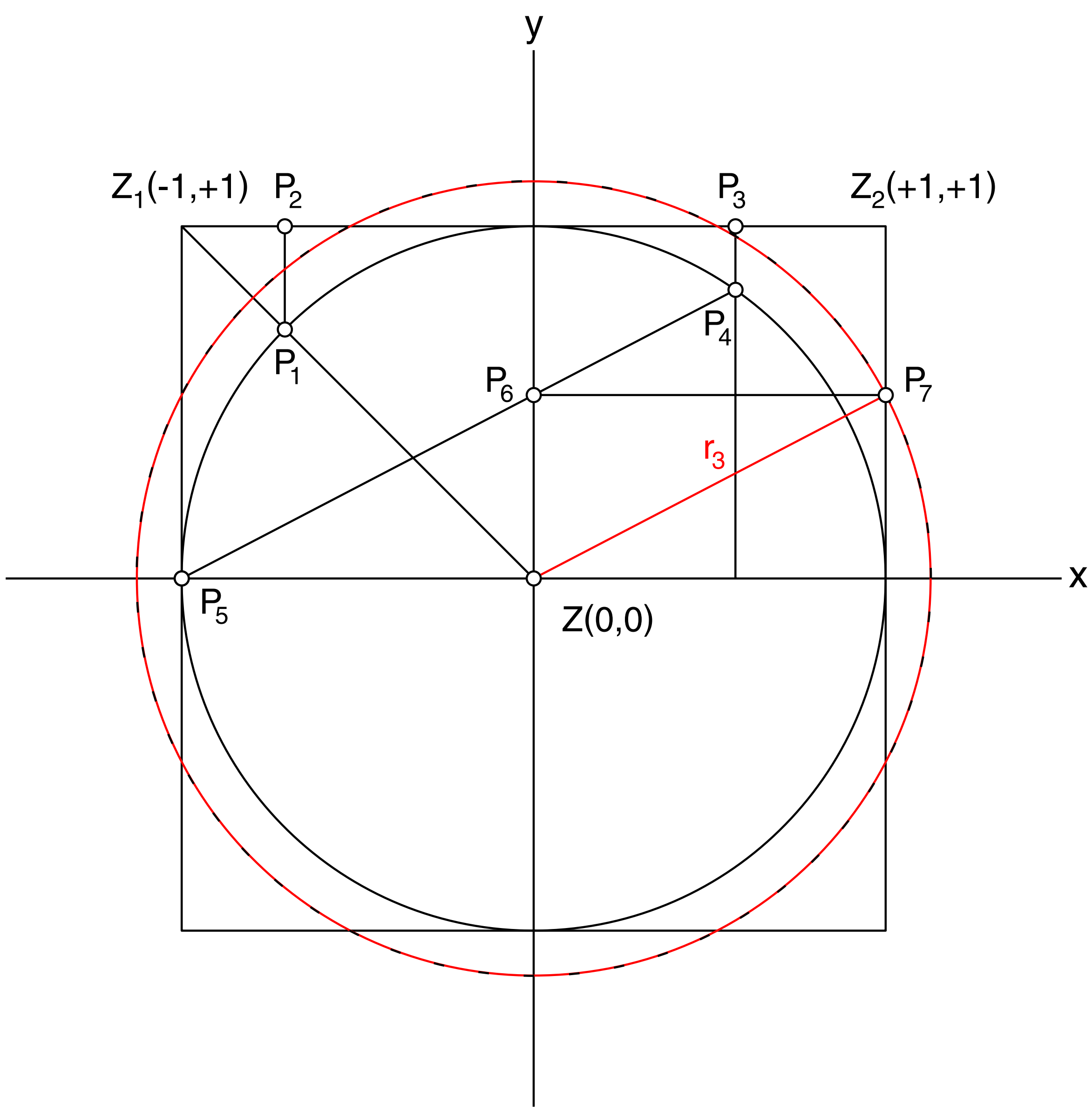
$$\pi' = 3.14644661 = (14 - \sqrt{2})/4$$

$$r_3 = \sqrt{16/(14-\sqrt{2})}$$

This is a good approximation and delivers the best solution so far. The drawing below is a little modified, but essentially based on the original author's ideas as published: a cute method how to establish π' by elementary geometrical constructions.

The square has the edge length two units.

1. Draw the black circle about Z
2. Draw the diagonal and find P_1
3. Find P_2 in vertical direction
4. Divide Z_2P_2 into four parts and find P_3 at the first quarter
5. Go down to P_4 on the black circle
6. Draw the line through P_4 to P_5
7. Find the intersection P_6
8. Project P_6 to P_7. The delivers the radius, which is again called r_3.



Square-Area / Circle-Area = 1.001545, equivalent to an accuracy of 0.15%

7. Method D / Mathematics

This proof (by G.Hoffmann) is not elegant but hopefully correct. For better readability the graphic is shown again.

$$x_1 = -\sqrt{2}/2$$
$$x_2 = x_1$$
$$x_3 = 1-(1+\sqrt{2}/2)/4 = (6-\sqrt{2})/8$$
$$x_4 = x_3$$
$$1+x_4 = (14-\sqrt{2})/8$$

Proportion

$$y_4/y_6 = (1+x_4)/1$$
$$y_6 = y_4/(1+x_4)$$

Circle

$$y_4^2 = 1-x_4^2$$

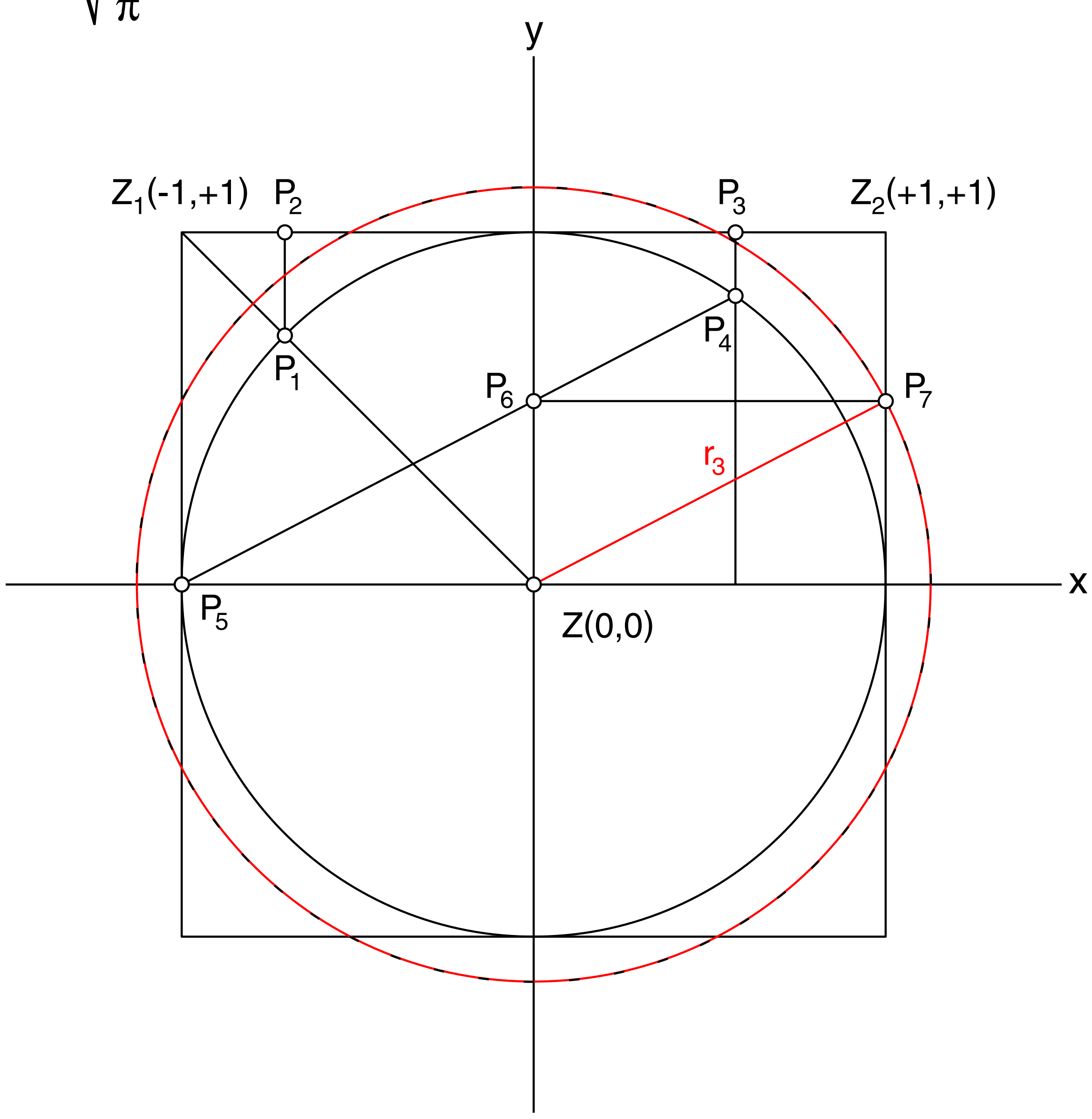
Pythagoras

$$r_3^2 = 1+y_6^2 = 1+y_4^2/(1+x_4)^2$$
$$= 1+(1-x_4^2)/(1+x_4)^2$$
$$= 2/(1+x_4)$$

$$r_3 = \sqrt{\frac{16}{14-\sqrt{2}}} = 2\sqrt{\frac{1}{\pi'}}$$

True radius

$$r = 2\sqrt{\frac{1}{\pi}}$$



8. References

[1] Volker Hoffmann
Der geometrische Entwurf der Hagia Sophia in Istanbul
Peter Lang AG
Moostrasse 1 / CH-2542 Pieterlen
info@peterlang.com
http://www.peterlang.net
In German, Turkish, English and French, 2005

[2] http://www.fho-emden.de/~hoffmann/hagiasophia.html

[3] Tan Tai Nguyen
http://www.dakhi.com/somen31.htm

[4] Reddivari Sarva Jagannadha Reddy
The untold story of the true value of π
sarvajagannadhareddy@yahoo.co.in

Title graphic by Nikolaos Theocharis

This document:
http://www.fho-emden.de/~hoffmann/circsqua22042005.pdf

Gernot Hoffmann
January 24 / 2006
Website
Load Browser / Click here