

Gernot Hoffmann

## Perceptually Optimized Grayscales

This document shows an attempt  
to find transfer functions for  
perceptually uniform Grayscales

Linear Transfer Function

Uncompensated Transfer Function

CIE Lab Conversion

Perceptual Optimization

# Perceptually Optimized Grayscale

A Grayscale is defined by e.g.  $n=64$  stepwise increasing values  $L_{sk}$  in the range of  $L_{sk} = 0.0 \dots 1.0$ . For  $n=64$  the increment is  $dL = 1/63$ . We do not use  $n=256$ , because then the differences between neighbours are not always perceivable.

The law is simply  $L_{sk+1} = L_{sk} + dL$  for  $k=0, \dots, 63$ , starting with  $L_{s0} = 0$ .

The color itself is primarily defined by  $C_k = R_k = G_k = B_k = \text{Round}(255 \cdot L_{sk})$ .

The rounding inaccuracy effect in color patches is avoided by dithering.

$C=102.4$  is represented by different pixels, 60% of  $C=102$  and 40% of  $C=103$ .

Since December 2007 all pages are lossless compressed (ZIP8).

## Important Note

Settings for Acrobat

Edit / Preferences / General /  
Page Display (since version 6)

Custom Resolution 72 dpi

Edit / Preferences / General /  
Color Management  
(full version only)

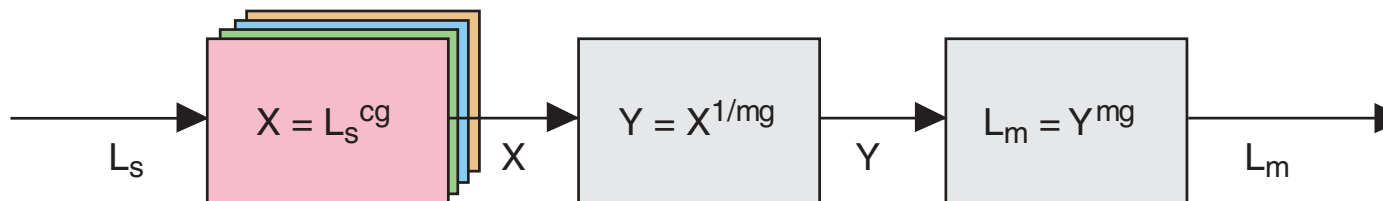
sRGB\*

EuroscaleCoated or  
ISOCoated or  
SWOP

GrayGamma 2.2

\*) The doc was made by pure power functions. Therefore a working space with exactly  $G=2.2$  without a linear slope would be more appropriate

$L_s$	Numerical data, linear light space, scene light
$X = L_s^{cg}$	Perceptual Correction
$Y = X^{1/mg}$	Monitor Gamma Compensation
$L_m = Y^{mg} = L_s^{cg}$	Monitor Luminance



## CIE Lab Lightness (index c for CIE Lab variables)

$$\begin{aligned} \text{If } Y_c > 0.008856 & \quad \text{Then } L_c^* = 1.16 \cdot Y_c^{1/3} - 0.16 & \quad \text{Else } L_c^* = 9.033 \cdot Y_c \\ \text{If } L_c^* > 0.008856 \cdot 9.033 & \quad \text{Then } Y_c = [(L_c^* + 0.16)/1.16]^3 & \quad \text{Else } Y_c = (1/9.033) \cdot L_c^* \end{aligned}$$

We use  $L_s = L_c^*$  as input and  $X = Y_c$  as output of the first block. For simplicity we can say, this is a correction law with  $cg = 3.0$ . For the test patches we use always the correct law, as above.

Of course the inverse monitor compensation has to be applied as well.

All examples are valid for Monitor-Gamma  $mg = 2.2$ .

Once again:  $cg$  describes the total transfer function from linear data to monitor luminance for an ideal calibrated monitor. CieLab is expected to show perceptually linear grayscales (wrong).

Case 1	$cg = 1.0$	Linear TransferFunction	Black background
Case 2			White background
Case 3	$cg = 2.2$	Uncompensated Transfer Function	Black background
Case 4			White background
Case 5	$cg = 3.0$	CieLab Mode	Black background
Case 6			White background
Case 7	$cg = 1.54$	Optimized Mode	Black background
Case 8			White background
Case 9			Gray background

## Resume 1

$L_s$  Luminance in a linear light space

$L_m$  Luminance of ideal monitor

Case 1+2 (page 4+5).  $L_m = L_s^{1.0}$

Linear Transfer Function looks too light

Linear doesn't show differences at the light end.

Linear image processing is good, but an output correction is necessary

Case 3+4 (page 6+7).  $L_m = L_s^{2.2}$

Uncompensated Transfer Function looks too dark at the dark end

Case 5+6 (page 8+9).  $L_m = L_s^{3.0}$

CieLab Mode looks too dark

CieLab on white background shows eight nearly equal grays the dark end.

CieLab is bad

Case 7-9 (page 10-13).  $L_m = L_s^{1.54}$

Optimized Mode shows reasonable resolutions at the dark and the light end

This is valid for black and white backgrounds

Accurate View:

Calibrated monitor

Gamma = 2.2

Text and line art smoothed

Images not smoothed

Zoom 200%

## Resume 2

A computer graphic, made by a linear light model:

This compensation has to be done by calculations:

Without any compensation	$c = 1.0$
With monitor compensation only	$c = 1.0/2.2 = 0.4545$
Optimized compensation	$c = 1.54/2.2 = 0.7$

A scan, as usual monitor compensated by 1/2.2:

This compensation has to be done by calculations:

With monitor compensation only	$c = 1.0$
Optimized compensation	$c = 1.54$

Further experiments showed some unexpected results:

The scanner menu indicated 2.2, the necessary compensation was  $c = 0.75$ . Just the opposite result. It can be assumed, that the scanner message means something like *compensated for Gamma=2.2 monitors*, but not necessarily by the expected inverse law.

## Nomenclature

### Compensation

Modification of an image by a power function.

Executed in one direction without a reverse operation.

As shown in the signal flow diagram on page 2, the monitor Gamma=2.2 is always already corrected.

### Gamma Working Space

*This is not discussed here.*

Linear light image data are distorted by a power function.

Calculations with these non-linear data are in principle wrong.

The reverse transformation is applied by the monitor.

Case 1

Linear Transfer Function

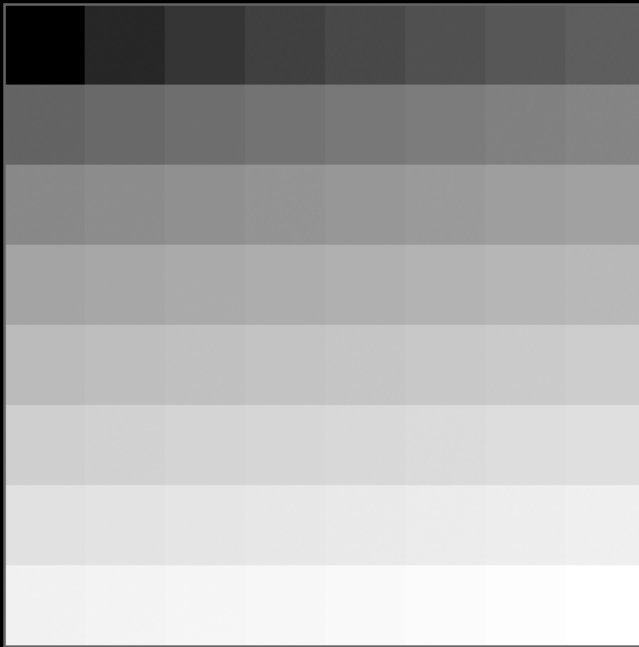
Black background

$$X = L_s^{1.0}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$

$$L_m = L_s^{1.0}$$



Case 2

Linear Transfer Function

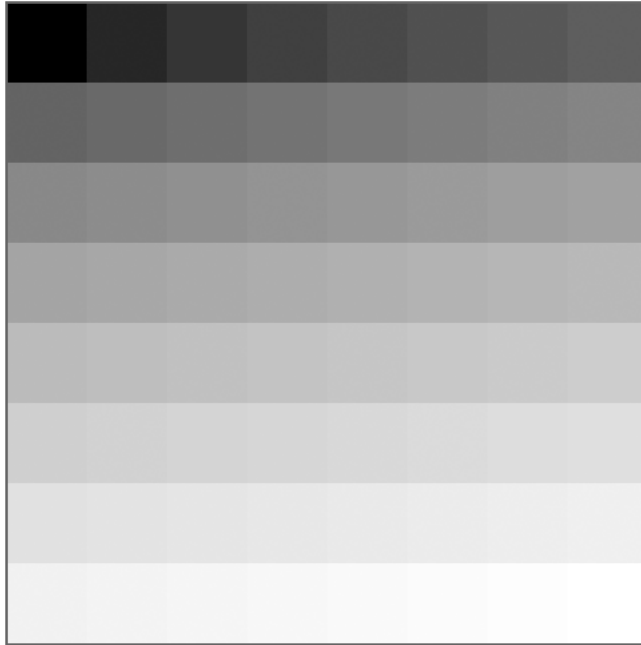
White background

$$X = L_s^{1.0}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$

$$L_m = L_s^{1.0}$$



Case 3

Uncompensated Transfer Function

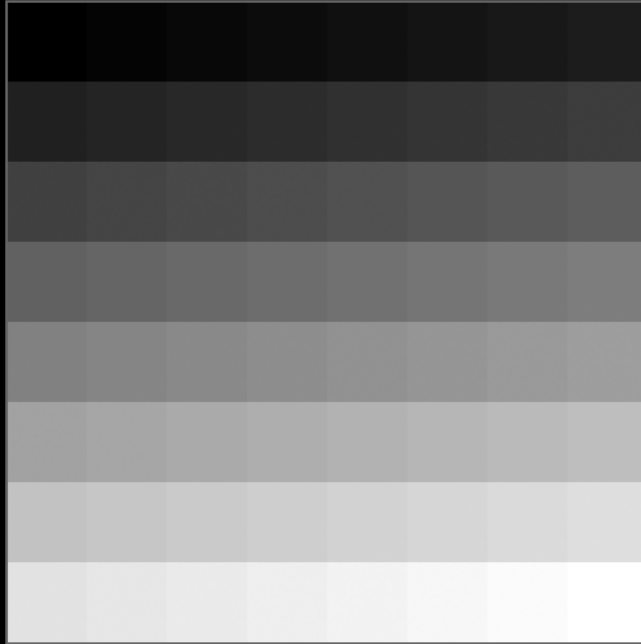
Black background

$$X = L_s^{2.2}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$

$$L_m = L_s^{2.2}$$



## Case 4

Uncompensated Transfer Function

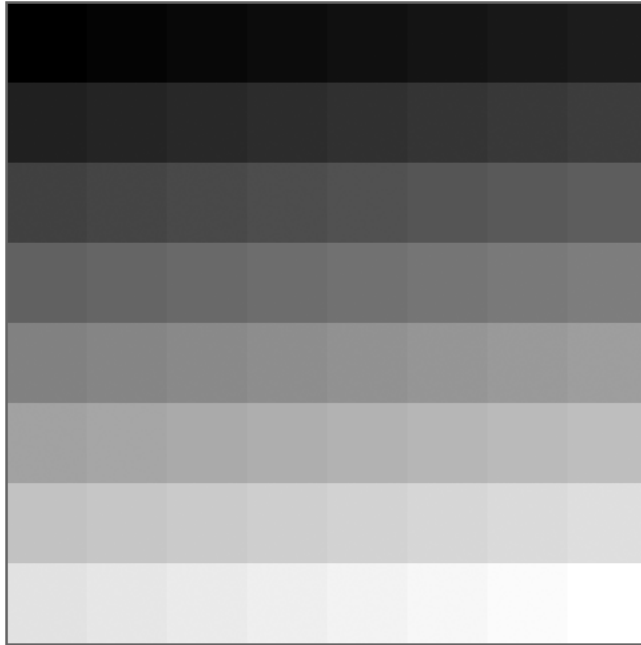
White background

$$X = L_s^{2.2}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$

$$L_m = L_s^{2.2}$$



Case 5

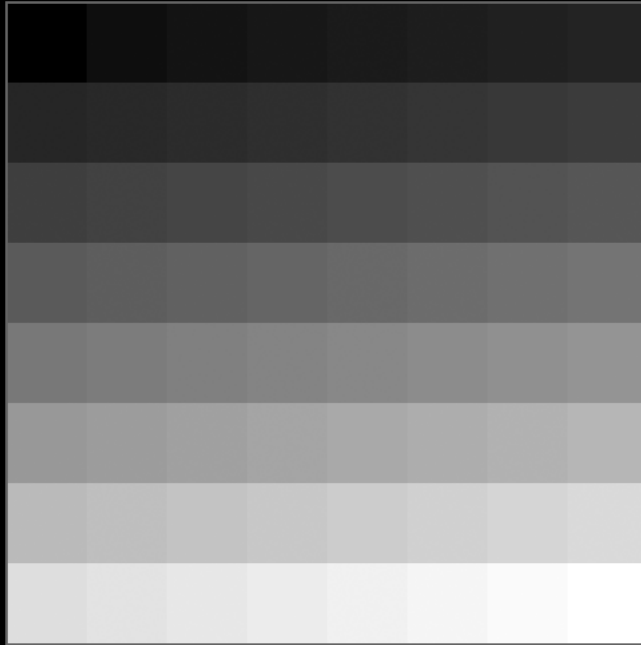
CIELab Mode

Black background

$$X = L_s^{3.0}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$



Case 6

CIELab Mode

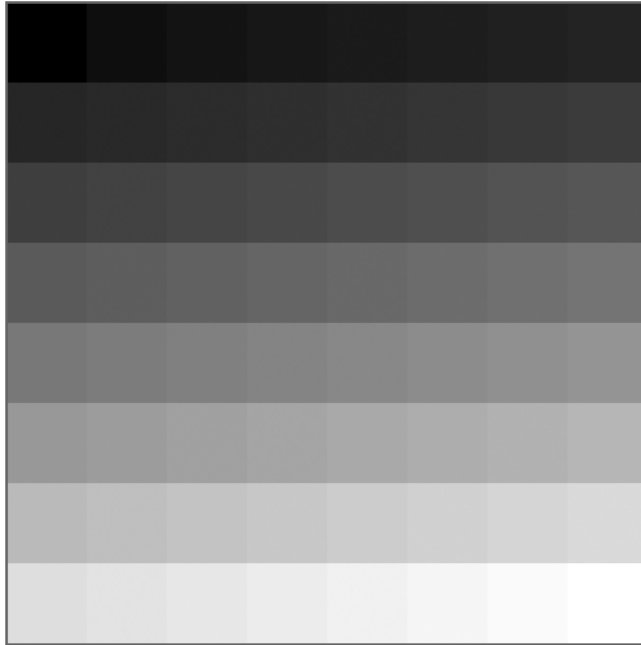
White background

$$X = L_s^{3.0} \text{ (CieLab)}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$

$$L_m = L_s^{3.0}$$



Case 7

Optimized Mode

Black background

$$X = L_s^{1.54}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$

$$L_m = L_s^{1.54}$$



Case 8

Optimized Mode

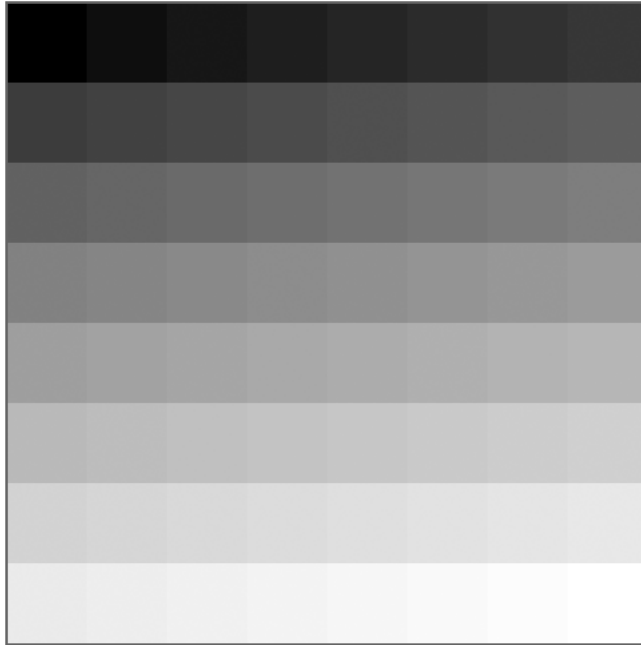
White background

$$X = L_s^{1.54}$$

$$Y = X^{1/2.2}$$

$$L_m = Y^{2.2}$$

$$L_m = L_s^{1.54}$$



Case 9

Optimized Mode

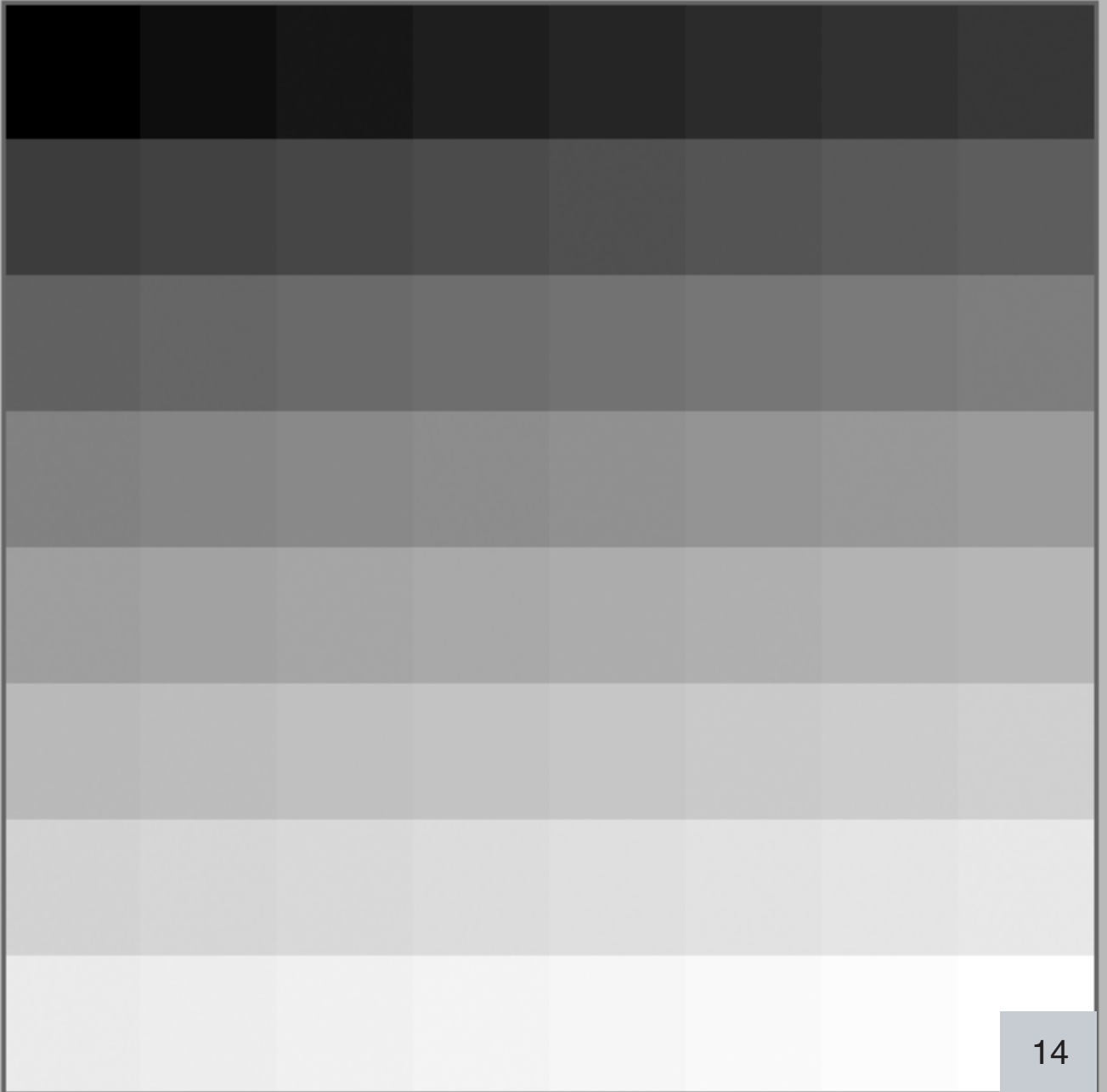
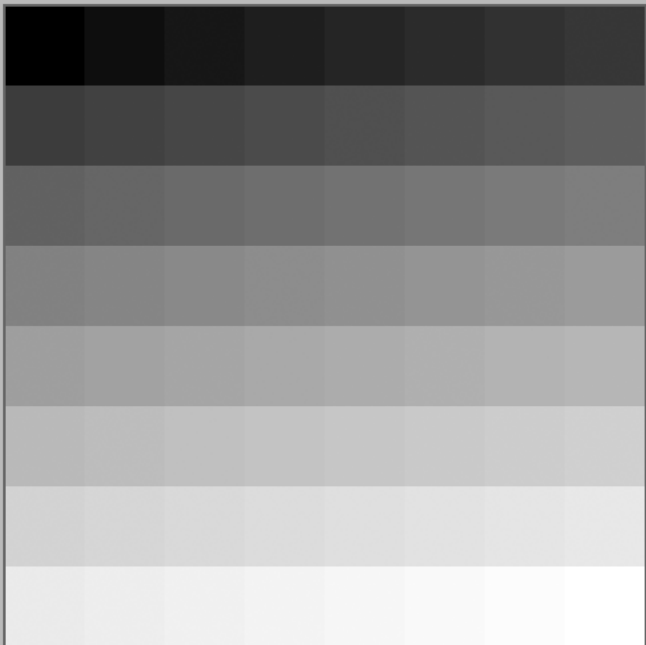
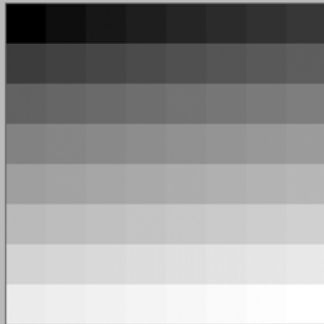
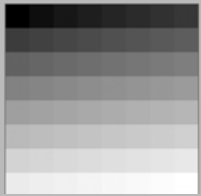
Gray background (186)

$$X = L_s^{1.54}$$

$$Y = X^{1/2.2}$$

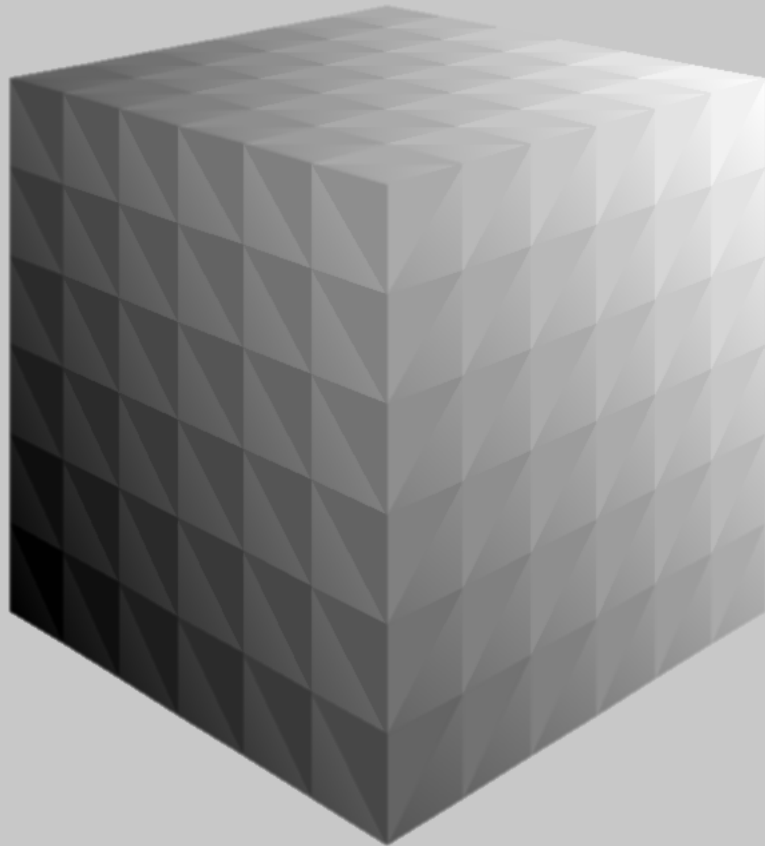
$$L_m = Y^{2.2}$$

$$L_m = L_s^{1.54}$$

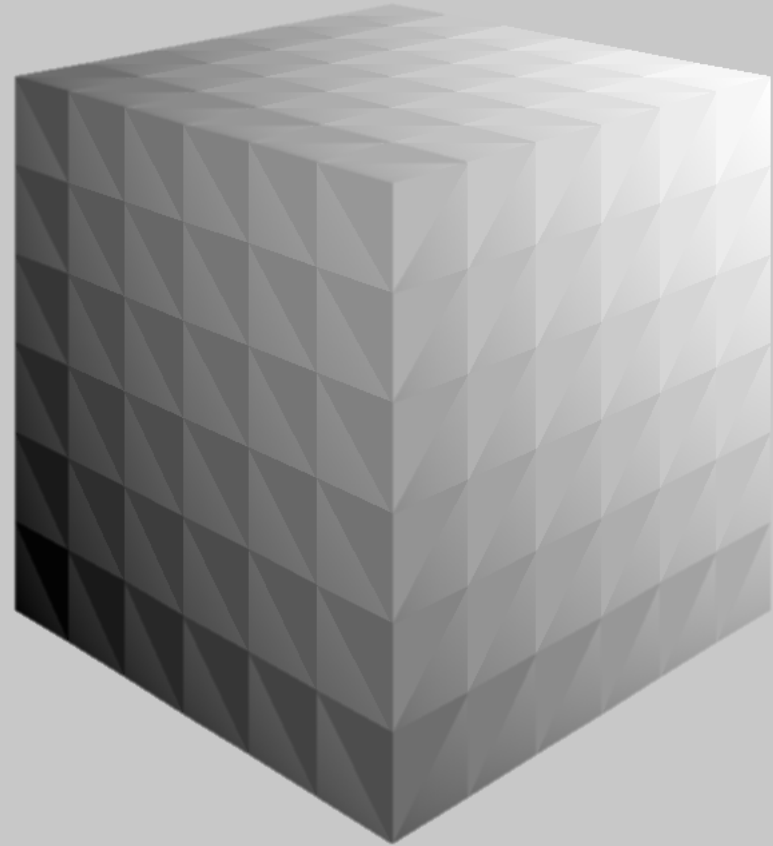




Color Cube with patterns  
Grayscale conversion  
Uncompensated  $L_m = L_s^{2.2}$   
Bad contrast for dark



Color Cube with patterns  
Grayscale conversion  
Optimized  $L_m = L_s^{1.54}$   
Better contrast for dark



# Final Test: Grayscale by a Different Program

$$L_m = L_s^{cg}$$



Uncompensated  
Linear Gray  
Distorted only by  
Monitor Gamma  
 $L_m = L_s^{2.2}$

### Test for Banding (1)

Pixel synchronized PDF  
Please use zoom 200%

Height 512 pixels. Centered at Gray 127/128.

Number of gray patches in one column

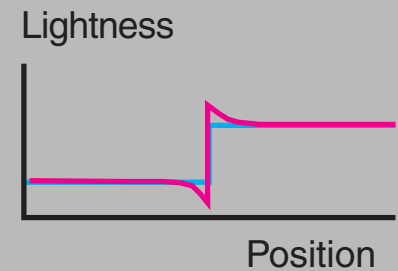
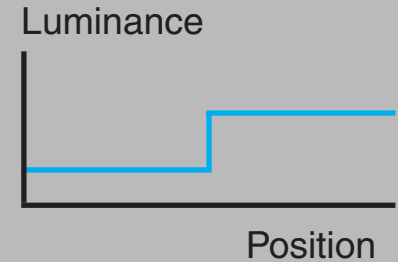
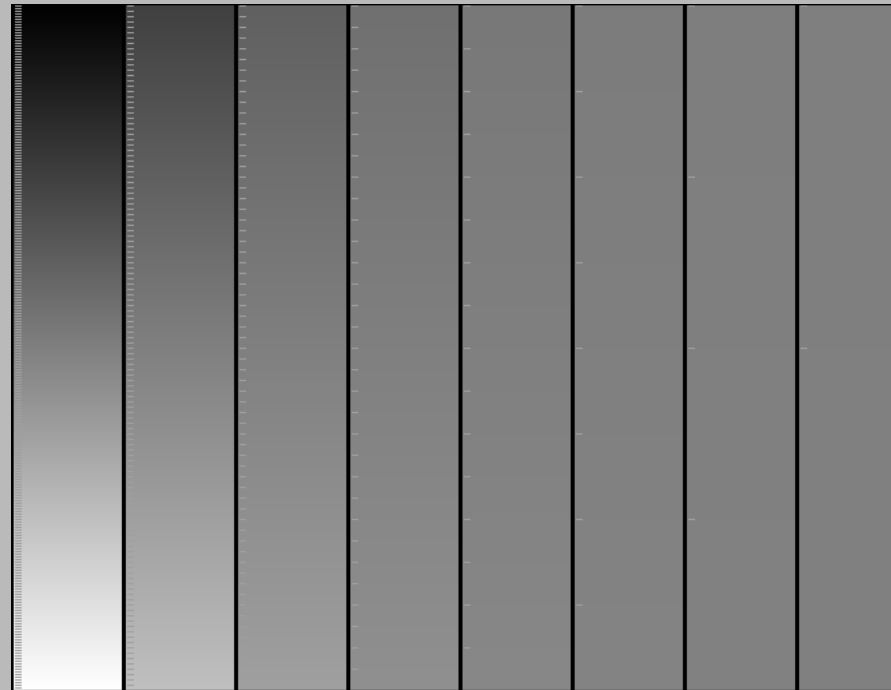
256 128 64 32 16 8 4 2

#### Mach Band Effect

Small transitions of the luminance can appear exaggerated as perceived lightness.

This kind of differentiation is caused by the eye's neuronal net by 'lateral inhibition'.

This feature is probably helpful for finding contours in real life images.



Uncompensated  
Linear Gray

Distorted only by  
Monitor Gamma

$$L_m = L_s^{2.2}$$

## Test for Banding (2)

Pixel synchronized PDF

Please use zoom 200%

Height and width 512 pixels

Lossless compression for pages 15-18

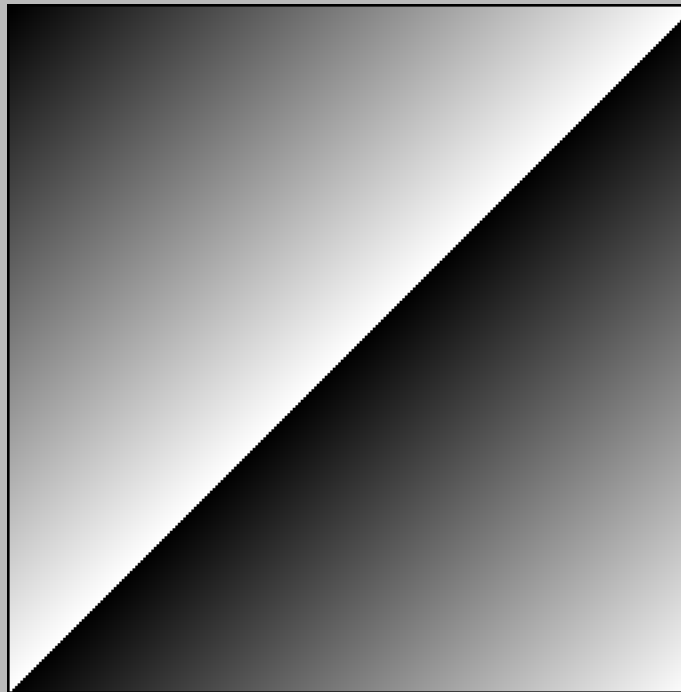
### Mach Band Effect

The effect is clearly visible on the pages 6 to 14, but not always visible on the previous page.

It depends on the monitor and the patch size.

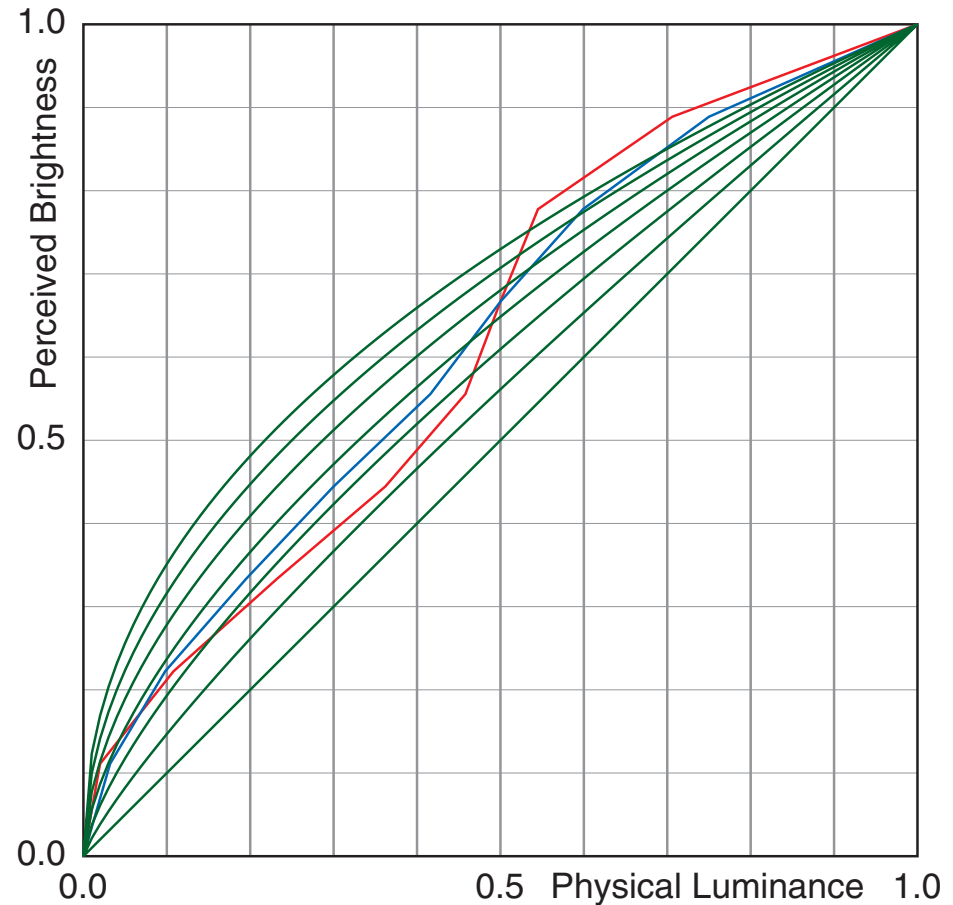
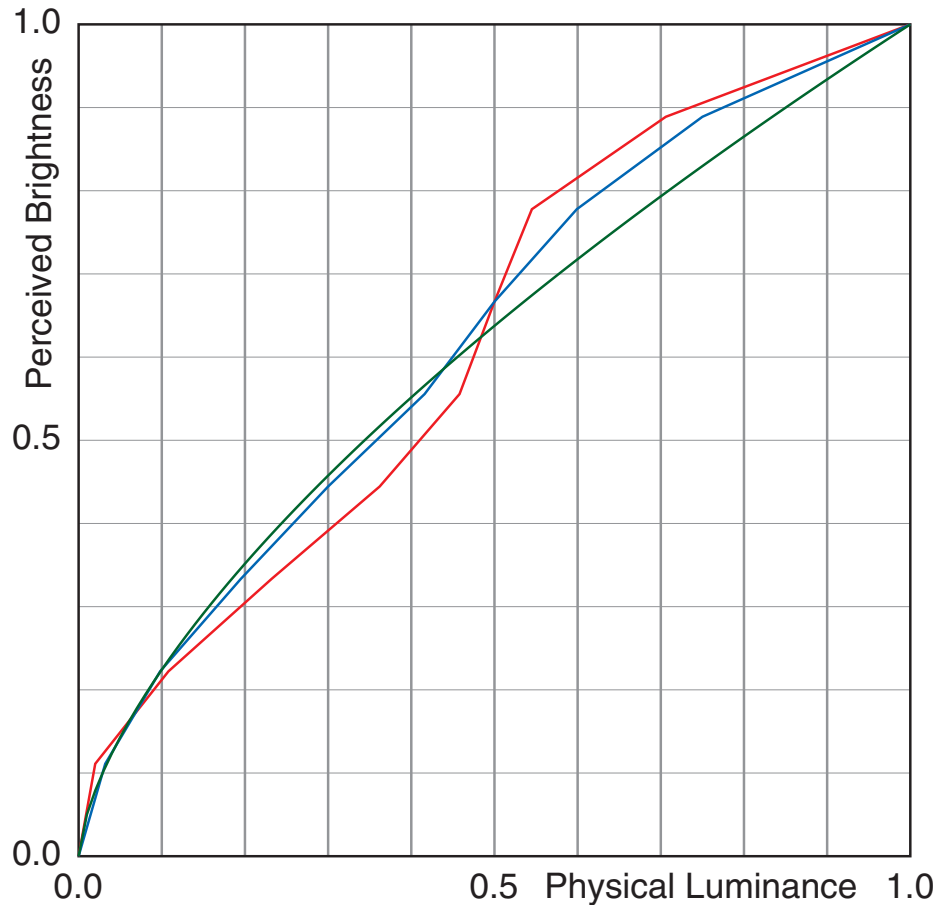
A 256 step grayscale is usually not perceived banded.

The monitor grid may cause artifacts too.



## Measured Brightness Perception

The diagrams are based on Fig.3 in [1]. The original doc shows absolute luminance from 0 to 111cd/m<sup>2</sup> and perceived brightness by 10 values (not 11). The normalized diagrams below show relative values from 0.0 to 1.0, stepsize 0.1. Therefore the corners of the polylines are not on the grid. The red curve is valid for patches on a flat medium gray background. The blue curve is valid for a similar test in a simulated threedimensional scene. The green curves were added by G.Hoffmann: In the left diagram a power function with exponent 1/1.54. In the right diagram power functions for 1/1.0 to 1/2.2 for steps 0.2.



## References

- [1] Surajit Nundy + Dale Purves  
A probabilistic explanation of brightness scaling  
14482-14487 / PNAS / October 29, 2002 / vol.99 / no.22  
<http://www.pnas.org/cgi/doi/10.1073/pnas.172520399>
  
- [2] Gernot Hoffmann  
The Gamma Question  
<http://www.fho-emden.de/~hoffmann/gamquest18102001.pdf>  
(more references)

This doc:

<http://www.fho-emden.de/~hoffmann/optigray06102001.pdf>